NOISE REDUCTION IN A MEDICAL IMAGING INSTRUMENT USING DISTRIBUTED PIEZOELECTRIC ACTUATOR/SENSOR BASED ON THE FEM MODELING

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Abstract
The purpose of this paper is reducing mechanical noise in a Magnetic Resonance Imaging (MRI) set. The vibration suppression in this instrument prevents the test results to be damaged, which is an important issue in medical tests. For this aim, the task of the structure modeling is tackled using the FEM approach and first order modal identification from the measured input-output data. The space model has been obtained from the dynamic frequency response of the modeled structure. The control problem of the funnel shaped structure is considered using LQG control method. Complete design and control development procedure is implemented in order to reduce the vibration magnitude of a funnel-shaped shell structure of the magnetic resonance tomography used in medical diagnostics.

1. INTRODUCTION

Medical imaging is the technique and process used to create images of the human body for clinical purposes or medical science. These developments are driven by the desire to obtain higher quality images that reveal more detail of the internal structure of the biological subjects [1].

A serious limiting factor in the development of these machines is the acoustic noise that they generate during scanning [2]. There are some sources for this vibration and acoustic noise, the main source of this is the gradient coil, which is used to produce a spatially varying dynamic magnetic field inside the MRI bore [3]. The high acoustic noise results in some problems for patients and workers such as annoyance to difficulties in verbal communication, heightened anxiety, temporary hearing loss and possible permanent hearing impairment for persons who are exposed to these noisy environments for long periods. The vibration of the gradient will also affect the image quality and resolution [4]. It is therefore important to find ways to reduce the vibration and acoustic noise levels of MRI scanners.

Due to its potential application in reducing interior cabin noise in MRI and other engineering structures, active control of structural vibration and acoustic has been a research area of growing interest for several years. Since low-frequency noise is directly coupled with the shell vibration [5,6], an alternative way of cabin noise suppression is to actively control the structural vibration of the shell and consequently reduce sound radiation into the cabin.

Piezoelectric sensing and control with distributed piezoelectric transducers have been intensively studied in some literatures [7,8,9,10].
The investigation of MRI gradient coil associated acoustic noise was first conducted in the late 1980s [11]. Later, extensive studies were performed using various MRI scanners and different pulse sequences [4,12,13,14,15]. In addition, Qiu and Tani studied the vibration control of a cylindrical shell used in MRI equipment [16]. Evans designed a vibration control method by using structural detuning and decoupling concept [17]. Yao et al. did a comprehensive investigation of the vibration properties of the coil insert was conducted using FEA and experimental testing [1].

This paper concerns the vibration control of the funnel-shaped inlet structure of the MRI tomograph using distributed piezoelectric actuator/sensor patches. The task of the structure modeling is tackled considering the FEM approach and model identification from the measured input-output data. Identification of the frequency response functions of the funnel as well as of the state space model using two kinds of subspace-based identification method is performed and the results are compared. The control problem of the funnel shaped structure is considered for two identified systems using LQG approach. Finally, results will be discussed.

2. MATERIALS AND METHODS

This part consists of the FEM formulation and modeling of the MRI inlet throat, the procedure of finding the optimal placement of the distributed piezoelectric actuators, identification of the FEM modeling using obtained input-output data and Linear Quadratic Gaussian controller design formulation.

2.1. FEM Modeling

Constitutive equations of piezoelectric material is given as

\[
T = C^E S - e^T E
D = eS + \varepsilon^E E
\]

In which \( T, S, E, D, C, e, \varepsilon \) is stress vector, deformation vector, electric field, electric displacement vector, constant elasticity matrix, constant dielectric matrix and piezoelectric coupling coefficients matrix, respectively. Dynamic equation of a smart structure is achievable using Hamilton principle in which Lagrangian and virtual work chosen appropriately. (2) shows the density of potential energy of piezoelectric material, which contains strain and electrostatic energy.

\[
H = \frac{1}{2} \left[ S^T T - E^T D \right]
\]

In addition, the virtual work is given as

\[
\partial W = \{ \delta u \}^T F - \delta \phi \sigma
\]

where \( F \) is applied force and \( \sigma \) is electric load. Using the variational principle for the piezoelectric material and substituting \( H \) and \( \partial W \) from (3) in (2) one can easily obtain

\[
0 = \int_{S_1} \rho \{ \delta u \}^T \{ \delta u \} - \{ \delta S \}^T C^E S + \{ \delta S \}^T e^T E + \{ \delta E \}^T e S + \{ \delta E \}^T \varepsilon^E E + \{ \delta u \}^T P_s dV
\]

\[
+ \int_{S_2} \{ \delta u \}^T P_c dS + \{ \delta u \}^T P_e - \int_{S_2} \delta \phi \sigma dS - \delta \phi Q
\]

In this finite element formulation \( u \) is displacement field and \( \phi \) is electric potential field. These fields can be described in displacement and potential of each node and this relation is established using mode shape functions which is shown as below.
\[
\{ u \} = [N_u] \{ u_i \} \\
\{ \phi \} = [N_\phi] \{ \phi_i \} 
\] 

Using (5) and (6), strain field and electric field can be obtained as

\[
\{ S \} = [D] [N_u] \{ u_i \} = [B_u] \{ u_i \} \\
\{ E \} = -\nabla [N_\phi] \{ \phi_i \} = -[B_\phi] \{ \phi_i \}
\] 

where \( \nabla \) is gradient operator and \( [D] \) is differential operator. By substituting (7) and (8) in (4) and using variational principle for any variation which satisfies the displacement and electric potential, the dynamic equations of piezoelectric media can be obtained as

\[
\{ M \} \{ \ddot{u}_i \} + [K_{uu}] \{ u_i \} + [K_{u\phi}] \{ \phi_i \} = \{ f_i \} \\
[K_{\phi u}] \{ u_i \} + [K_{\phi\phi}] \{ \phi_i \} = \{ g_i \}
\]

where

\[
[M] = \int_V \rho [N_u]^T [N_u] dV \\
[K_{uu}] = \int_V [B_u]^T C^E [B_u] dV \\
[K_{u\phi}] = \int_V [B_u]^T e^T [B_\phi] dV \\
[K_{\phi u}] = [K_{u\phi}] \\
[K_{\phi\phi}] = \int_V [B_\phi]^T c[B_\phi] dV
\]

And

\[
\{ f_i \} = \int_V [N_u]^T \{ P_u \} dV + \int_{s_1} [N_u]^T \{ P_s \} ds + [N_u]^T \{ P_e \} \\
\{ g_i \} = \int_{s_2} [N_\phi]^T \sigma ds - [N_\phi]^T Q
\]

Equations (11) and (12) introduce mass matrix, stiffness matrix, piezoelectric coupling matrix, capacitance matrix, mechanical force and electrical load, respectively. After introducing the finite element formulation and parameters, ANSYS is used to apply this formulation.

### 2.2. Optimal Placement

The Actuation Authority is selected as the optimization function. In the case of considering more than one mode shape, this function considers amplitudes around the natural frequencies of the same mode shape by weighting coefficients. The maximum frequency response of the system is equivalent to the maximum displacement which caused by the piezoelectric actuator. Before starting the optimization procedure, the entire possible placement of the piezoelectric actuator is considered. As shown in the Fig. 1, the structure contains two parts of flat and curved section. By considering axisymmetricity, 56 places are available to locate the actuators (Fig. 2). Because of symmetric shape of the throat, actuators locations are chosen as Fig. 2 and no optimization is done.
Genetic algorithms (GA) have been proved to suitably work on a wide range of optimization problems from different fields. However, for finding the optimal answer of this optimization problem a nonlinear optimization problem to be solved. Furthermore in this case this gene is m-dimensional string in which m is the number of the optimal actuators and it has to be chosen by the user. This string contains a real positive non-recurring numbers starting from 1 to 56 and it is obvious that each string presents a set of piezoelectric actuators. For this optimization problem that rectifies the last one, two new indexes is added to the consideration. First

\begin{equation}
\text{Minimum Effectiveness (ME)} = \frac{\text{Actuation authority of actuator with lowest authority}}{\text{Actuation authority of actuator with highest authority}} \sum \text{Authority of all actuation in group} \noindent \text{No. of Actuators} \noindent \text{Actuation authority of actuator with highest authority}
\end{equation}

\begin{equation}
\text{Relative Mean Effectiveness (RME)} = \frac{\text{Actuation authority of actuator with highest authority}}{\sum \text{Authority of all actuation in group}} \noindent \text{No. of Actuators} \noindent \text{Actuation authority of actuator with highest authority}
\end{equation}

It is obvious that the effectiveness of the actuation of the sets with larger number of actuators is more than others. However, this increasing of the effectiveness itself saturates as the number of actuators increases.

### 2.3. System Identification

After extracting the frequency response function of the structure, for controlling purposes, a suitable transfer function that can describe the closest response to FEM model should be found. A system identification method based on first order modal analysis will be used in this paper. In this method the identified frequency response can be described as

\begin{equation}
\alpha_{jk} (\omega) = \frac{r A_{jk}}{\omega_r^2 - \omega^2 + j \eta \omega \omega_r} \sum_{\sigma_r} \frac{r A_{jk}}{\omega_r^2 - \omega^2 + i \eta \omega \omega_r} + \frac{r A_{jk}}{\omega_r^2 - \omega^2 + i \eta \omega \omega_r} + \frac{r B_{jk}}{\omega_r^2 - \omega^2 + i \eta \omega \omega_r} + B_{jk}
\end{equation}

where \( N \) is number of mode shapes in consideration, \( r A_{jk} \) is the modal constant which is obtained from the \( \phi_{jr} \), that introduces the \( j \)th element in the \( r \)th Eigen vector, \( r A_{jk} = (\phi_{jr})(\phi_{jr}) \), \( \omega_r \) is the resonance frequency and \( \omega \) is a frequency around the resonance frequency. Also, the peak picking method is used for system identification.

### 2.4. Controller Design

Before designing the optimal controller, a MIMO state-space representation based on the obtained transfer function should be considered. Accordingly, following the standard procedure outlined by Gawronski [18], the state space realization of four obtained transfer function from each identification method, can be written as

\begin{equation}
\dot{z}(t) = A_m z(t) + B_m u(t) + B_n w_d
\end{equation}

\begin{equation}
y(t) = C_m z(t) + w_n
\end{equation}

where \( B_w \) is disturbance matrix, \( w_n \) is the measurement noise, \( w_d \) is associated with the disturbance signals (i.e., it may include actuator noise and/or any external random disturbance), and obtaining the modal state-space representation triple \((A_m, B_m, C_m)\) requires some relatively tedious manipulations, which is explained by Maciejowski [19]. Also, the measurement noise, \( w_n \), and the process noise, \( w_d \), are generally assumed to be uncorrelated zero-mean Gaussian stochastic processes with constant power spectral density matrices, \( W_n \) and \( W_d \), respectively.

The linear-quadratic (LQ) state-feedback regulator with output weighting problem, where all the states are known, is the deterministic initial value problem by minimizing the cost function.
\[ J = \int_0^\infty \left( y(t)^T Q y(t) + u(t)^T R u(t) \right) dt, \quad (17) \]

where \( Q \) and \( R \) are suitably chosen constant weighting matrices, such that \( Q = Q^T \geq 0 \), and \( R = R^T > 0 \).

The first step in the solution of the LQG problem consists of finding the optimal control to a deterministic linear quadratic regulator (LQR) problem: namely, the above LQG problem without \( w_d \) and \( w_n \). The optimal solution to this problem can be written in terms of the simple state feedback law \[ u(t) = -K_c \hat{z}(t), \quad (18) \]

where \( \hat{z} \) is the estimated state, and \( K_c = R^{-1}B_m^T X \), in which \( X (X = X^T \geq 0) \) is the unique positive-semi definite solution of the algebraic Riccati equation

\[ A_m^T X + X A_m - X B_m R^{-1} B_m^T X + Q = 0. \quad (19) \]

The next step is to find an optimal estimate \( \hat{z} \) of the state \( z \), so that \( E \left[ (z - \hat{z})^T (z - \hat{z}) \right] \) is minimized. The optimal state estimate is given by a Kalman filter, which estimates the state of the system in presence of noisy measurements, and is independent of \( Q \) and \( R \). The Kalman filter has the structure of an ordinary state estimator or observer with

\[ \hat{z}(t) = A_m \hat{z}(t) + B_m u(t) + K_e \left[ y(t) - C_m \hat{z}(t) \right], \quad (20) \]

where \( K_e = Y C_m^T W^{-1}_m \) is the optimal choice for observer gain which minimizes the mean square error \( E \left[ (z - \hat{z})^T (z - \hat{z}) \right] \), and \( Y = Y^T \geq 0 \) is the unique positive-semi definite solution of the estimator algebraic Riccati equation \[ Y A_m^T + A_m Y - Y C_m^T V^{-1} C_m Y + W_d = 0. \quad (21) \]

Lastly, after some straightforward manipulations, one can obtain the closed-loop system dynamic equations in the form

\[
\frac{d}{dt} \begin{bmatrix} z(t) \\ e(t) \end{bmatrix} = \begin{bmatrix} A_m - B_m K_e & B_m K_c \\ 0 & A_m - K_e C_m \end{bmatrix} \begin{bmatrix} z(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} B_m w_d \\ B_n w_d - K_e w_n \end{bmatrix}, \quad (22)
\]

where \( e(t) = z(t) - \hat{z}(t) \). Next, some numerical examples will be considered.

3. RESULTS

In order to model piezoelectric in ANSYS, one can use coupled field analysis and coupled elements. Figure 1 shows the entrance of a MRI. Because of the complex shape of the MRI throat, the dynamic exact model of this structure is not practical so finite element method is used to achieve the dynamic model. The MRI throat is made of Aluminum. Furthermore, the material characteristic for piezoelectric sensor/actuator is PZT4.
Table 1 shows the comparison of the natural frequencies. As one can see, this comparison shows an appropriate accommodation between these two methods, so the FEM model is reliable and accurate enough.

<table>
<thead>
<tr>
<th>No. of Mode shape</th>
<th>Natural Frequency [17]</th>
<th>Natural Frequency (FEM)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.573</td>
<td>9.398</td>
<td>1.82</td>
</tr>
<tr>
<td>2</td>
<td>23.333</td>
<td>22.434</td>
<td>3.85</td>
</tr>
<tr>
<td>3</td>
<td>31.439</td>
<td>31.806</td>
<td>1.16</td>
</tr>
</tbody>
</table>

The characteristics of used genetic algorithm in the process are as follows: Population size = 10, Crossover fraction=0.8, Migration fraction=0.2, max iteration=100. For considering the optimization solutions, the Actuation Authority of three mode shapes considered as nominal system also the optimization problem considered for 1 actuator up to 5 actuators. The optimization results for 3 first mode shapes are listed in Table 2.

<table>
<thead>
<tr>
<th>RME</th>
<th>ME</th>
<th>Actuation Authority $\times 10^{-2}$ (mm)</th>
<th>Selected Set</th>
<th>No. of Actuators</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4.82</td>
<td>42</td>
<td>1</td>
</tr>
<tr>
<td>0.876</td>
<td>0.751</td>
<td>8.44</td>
<td>42, 56</td>
<td>2</td>
</tr>
<tr>
<td>0.834</td>
<td>0.75</td>
<td>12.06</td>
<td>42, 55, 56</td>
<td>3</td>
</tr>
<tr>
<td>0.741</td>
<td>0.461</td>
<td>14.28</td>
<td>23, 42, 55, 56</td>
<td>4</td>
</tr>
<tr>
<td>0.675</td>
<td>0.412</td>
<td>16.26</td>
<td>23, 38, 42, 45, 56</td>
<td>5</td>
</tr>
<tr>
<td>0.471</td>
<td>0.205</td>
<td>22.71</td>
<td>4, 12, 23, 29, 32, 38, 42, 45, 55, 56</td>
<td>10</td>
</tr>
<tr>
<td>0.01</td>
<td>0.004</td>
<td>$2.5 \times 10^{-1}$</td>
<td>1, 2, 9, 14, 17</td>
<td>Worst set of 5 Actuators</td>
</tr>
</tbody>
</table>

As one can see in Table 2, the RME index decreases as the numbers of the actuators increase, this index for 5 actuators decreases to 67% and as results show after the third actuator the RME has a sudden drop and it can easily seen that in this case the first actuator has only 46% performance of the first actuator.
Two groups of 6 actuators optimal placements are shown in Fig. 3. These actuators divided in two groups. Note that the piezoelectric sensors are located on two fixed location without any optimization. This let us design a controller with multi inputs and outputs.

Using first order method for both first group and second group of actuators, 4 transfer functions from each group of actuators to the each one of the sensors to be found. The dynamic response and the identified system using first order identification method are shown in Fig. 4.

In order to suppress the vibration of the structure an optimal controller is designed with the procedure introduced in the last section. The comparison of the frequency responses of the open loop system and the closed-loop system identified by first order method (using designed controller by a specified $Q$ and $R$) is shown in Fig. 5.

Note that the disturbances are applied from the control channels. This figure shows the effectiveness of the proposed controller to decrease the amplitude of the sensors voltages, especially near the natural frequencies. Comparison of the open loop system and closed-loop first order identified system against a random input disturbance is indicated in Fig. 6. This figure shows the appropriate performance of the designed controller to reject the random disturbances. Figure 7 shows the actuator voltages as control input during applying random disturbance to the control input channels.
Fig. 5. Frequency responses of the open loop and the closed-loop system.

Fig. 6. The open loop and the closed-loop system responses to random disturbances.

Fig. 7. Control inputs the open loop and the closed-loop system responses to random disturbances.
4. CONCLUSION

In this paper, the optimal LQG system was proposed as a solution for the control of vibrations caused by disturbances, in the sense of the vibration magnitudes suppression. The controller is aimed at active vibration control of funnel shaped structures with distributed piezoelectric actuators and sensors, but it can be widely used for different control tasks. Since the control design approach is a model based one, an approach of state space model identification has been considered in this paper: first order modal analysis. Proposed controller is implemented on the funnel shaped model, the inlet part of the MRI tomography, considering optimal placement the actuators and sensors.

The results showed considerable reduction of the vibration magnitudes in all frequency range. For investigation of the designed controller performance in time domain, closed-loop systems responses due to random disturbance were compared with the open loop ones.

As it is clearly shown in the results, the active vibration attenuation has done great job in suppressing vibration amplitude and resultant acoustic noise which is a crucial problem in applied medical instrumentation.

REFERENCES
