Error Compensation for Continuous Path Operation of 3-RRR Planar Parallel Manipulator with Clearance in the Joints

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ABSTRACT

In this paper, we study continuous path operation of a 3-RRR planar parallel manipulator whose joints have clearance. During continuous path operation end-effector must go through a predefined path. However, clearance introduces an additional uncontrollable degree of freedom to the manipulator. Therefore it causes error and should not be neglected in a successful path operation. In this paper, inverse and direct kinematics- with and without considering clearances, of the manipulator have been utilized. The direct kinematics in the presence of clearance is complicated and has no analytical solution. Therefore, a Neural Network is used to tackle this problem. The clearances are modelled as mass-less virtual links whose angular displacement are obtained from dynamic analysis of the manipulator. We have shown that the errors in the path can be efficiently compensated by appropriate modifications of the inputs for both vertical and horizontal planes. Perhaps surprisingly, the flexibility and generality of the proposed method makes it applicable to any parallel manipulator with revolute joints by just modifying the actuator inputs and without any changes in the design of manipulator.

Keywords: Parallel manipulator, continuous path operation, clearance, dynamics, kinematics

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1 INTRODUCTION

Robotic manipulators have been widely used in advanced manufacturing systems, in which positioning accuracy and dynamic performance are highly determinative. There are many uncertain factors influencing the accuracy and repeatability of robotic manipulators. In the conventional kinematic and dynamic analysis of robotic manipulator systems, each joint is characterized as a perfect adjustment; i.e., clearance, wear and manufacturing tolerances are not considered. However, clearances in the joints are inevitable due to tolerances and defects arising from manufacturing and design. Therefore, this issue has attracted the attention of researchers recently [1-4]. Yan and Guo [5] studied kinematic accuracy analysis of flexible mechanisms with uncertain link length and joint clearance. They used impact-pair model to describe the separation between two components in the joints with clearance. Bu et al. [6] adopted dynamic analysis to select the appropriate path parameters in order to keep joint elements attached in a path planning problem. Erkaya and Uzmay [7] studied the effect of clearances on a planar four-bar mechanism path generation. They aimed minimizing the relative error between the target and real paths which is tackled using optimization over link arguments. To do so they utilize the Genetic Algorithms framework to find the direction of the clearances. Altuzarra et al. [8] studied the clearance effects in parallel manipulators and presented a numerical procedure for the accuracy analysis. Sardashti et al. [9] investigated optimal free-defect function generation synthesis of for-bar linkage with joint clearance using PSO algorithm. They used continuous contact model for joint clearances in their study. Wu et al. [10] modeled a planar 3-PPR parallel manipulator with joint clearance and performed an experimental validation for the error, as well. They modeled clearance in both prismatic and revolute joints. For revolute joints, they used continuous contact model and virtual link to model the joint clearance. Moreover, Wu and Rao [11,12] used value range computation to find the optimum solution considering manufacturing expenses.

Many researchers focused on the dynamic analysis of manipulators, as well [13,14]. Flores et al. [15] utilized a continuous contact approach for revolute joints with clearance in order to study the dynamics of two planar mechanisms including four-bar and slider-crank mechanisms. Zhe et al. [16,17] proposed a criterion for the detachment of joint elements based on continuous contact model. Zhang et al. [18] studied the dynamical properties of 3-RRR planner manipulator with the assumption that it has a number of joints with clearance. They have shown that the clearance affects the acceleration, speed, and displacement of the mobile components and its driving moments dramatically. Xu et al. [19] investigated dynamic analysis of a slider-crank mechanism with two types of non-ideal revolute joints. They presented an approach for dynamic modeling of a multi-body system with two types of bearing including rolling element bearing and journal bearing, and used a slider-crank mechanism to demonstrate the efficiency of their method.

The joint clearance does not guarantee no abrupt motion change happens. Also, impulsive effect will be generated by the contact forces in the presence of joint clearance. Many researchers studied motion and path planning of parallel manipulators. Angeles et al. [20] derive a formulation of the trajectory planning for continuous-path operation in the configuration space. Rubio et al. [21] studied the optimal time trajectories for industrial robots with torque, power, jerk and energy consumed constraints. They tried to choose the most efficient way of working and found a relationship between the maximum and minimum values of the parameters under study for a couple of examples.

Sivakumar et al. [22] investigated the possibility of applying Genetic Algorithms for path planning of construction manipulators. Parsa et al. [23] used an approach based on Neural Networks to find the solutions of direct kinematic equations and planned a trajectory to avoid singularities and obstacles. Farajtabar et al. [24] performed the pick and place trajectory planning of a 3-RRR planar parallel manipulator with 3 joints with clearance which were attached to the end-effector. They used a 3-4-5 polynomial for trajectory, in which, the input motors have smooth changes in acceleration and torque. In their work the end-effector had no necessity to follow a specified path, but the start and end points were important. Our work differs from theirs in the sense that all un-actuated joints are considered to have
clearance, and this makes the inverse and direct kinematic equations different and more complicated, and a properly engineered Neural Network is needed for solving the direct kinematics. Furthermore, in the present work the end-effector must follow a specified path and this limitation makes the problem more important form applicability point of view. In this study, we propose to compensate for the error in the continuous path operation problem by appropriate modifications of the inputs. We make the assumption that the framework obeys continues contact criteria in which the direction of force vectors coincides with the line perpendicular to the tangent plane of journal and bearing [1,3,5-7,24].

The forces are determined by the Newton-Euler equations. We illustrate the efficiency of the algorithm with two numerical examples. The method is general and works for any manipulator with any running speed and payload. As another contribution, we tackle the direct kinematics problem using a novel Neural Networks based approach. A system of nonlinear equations is the result of the direct kinematics problem posed by the mechanism with clearance. The problem is solved in the vertical plane, as well as in the horizontal plane, to take the gravity effect into account.

The paper proceeds as follows. In section 2, the general mechanism and the model of clearance are introduced. The analysis of the manipulator with clearance is included in the 3rd section. Moreover, inverse and direct kinematics and dynamic analysis of the ideal and non-ideal mechanisms are performed. Section 4 presents the proposed method for continuous path operation of the manipulator. Then the 5th section contains the results and finally, the paper is concluded in section 6 accompanied with a discussion.

2 GENERAL 3-RRR PLANAR PARALLEL MANIPULATOR

The 3-DOF 3-RRR planar parallel manipulator, depicted in Fig. 1, consists of a kinematic chain with three legs, which connect the end-effector to the base with nine revolute joints. Moreover, three motors are fixed to the base at \( M_1, \ M_2, \ M_3 \).

It is assumed that six joints \( A_1 \), \( A_2 \), \( A_3 \), \( B_1 \), \( B_2 \) and \( B_3 \) have clearance (Fig. 2(a)). It is noted that, Staircase-shaped velocities and high acceleration are results of impact forces. Lubricating joins will prevent these phenomena. Therefore, continuous contact model with low friction due to lubrication is logical for such a joint [1,24,25].

![General 3-RRR planar parallel manipulator.](image)

Fig. 1. General 3-RRR planar parallel manipulator.
In continuous contact model, the deviation of the radii of the pin and the socket is defined as clearance, which can be modeled as a mass-less virtual link of the fixed length [1,3,5-7,24]. This is demonstrated in Fig. 2(b). Zhe et al. [16,17] showed that the direction of the forces in the ideal joints is an appropriate approximation for measuring the position of the mass-less clearance links in the corresponding ideal joints.

![Fig. 2. (a) Non-ideal mechanism. (b) the model of clearance.](image)

3 MANIPULATOR ANALYSIS IN THE PRESENCE OF CLEARANCE

3.1 Inverse Kinematics

The mechanism with clearance at $A_i$ and $B_i$ (i=1,2,3) is illustrated in Fig. 2(a). Inverse kinematics is the process of finding actuator variables for a given position of the end-effector. The elbow-up and the elbow-down are the two solutions of the inverse kinematics problem for each leg of the conventional manipulator. Therefore, there are eight solutions for the problem of which one has been considered in the present work [23]

The configuration of leg i is depicted in Fig. 3. Accordingly, the inverse kinematic equations are as follows:

$$\theta_i = \alpha_i \pm \psi_i \quad (i=1,2,3)$$

where:
\[ \alpha_i = \tan^{-1}\left(\frac{(y_{2i} - y_{0i})}{(x_{2i} - x_{0i})}\right) \]  

(2)

\[ \sin \sin( \alpha_i) - \sin( \beta_i) = 0 \]  

(5)

where \( \phi_i = \phi + \pi/6 \), \( \phi_2 = \phi + 5\pi/6 \) and \( \beta_3 = \phi - \pi/2 \). The bases of the manipulator are given as: \( x_{01} = 0 \), \( x_{02} = 1 \), \( x_{03} = 1/2 \) and \( y_{01} = 0 \), \( y_{02} = 0 \), \( y_{03} = \sqrt{3}/2 \). Furthermore, \( \gamma_i \) is the angular displacement of the end-effector and \( (x, y) \) is its coordinate. Considering the chain \( MB_i'B_i'A_i \) as a four-bar linkage, one can write kinematic equations both in perpendicular and parallel to \( M_iA_i \), i.e.,

\[ l_i \cos \psi_i + r \cos(\delta_i - \alpha_i) + l_2 \cos(\beta_i - \alpha_i) = P_i \]  

(4)

\[ l_i \sin \psi_i + r \sin(\delta_i - \alpha_i) + l_2 \sin(\beta_i - \alpha_i) = 0 \]  

(5)

where \( P_i = \sqrt{(x_{2i} - x_{0i})^2 + (y_{2i} - y_{0i})^2} \), and \( \delta_i \) is the clearance angle in joint \( B_i \).

Squaring Eqs. (4) and (5), adding them up and eliminating \( \beta_i \) lead to:

\[ k_i + k_2 \cos \psi_i + k_3 \cos(\delta_i - \alpha_i) = \cos \psi_i \cos(\delta_i - \alpha_i) + \sin \psi_i \sin(\delta_i - \alpha_i) \]  

(6)
Where, \( k_1 = (l_2^2 - l_1^2 - r^2 - P^2) / 2r l_1 \), \( k_2 = P / r \) and \( k_3 = P / l_1 \). By appropriate change of variables, i.e., \( T = \tan(\psi_i / 2) \), \( \sin \psi_i = (2T) / (1 + T^2) \) and \( \cos \psi_i = (1 - T^2) / (1 + T^2) \), Eq. 6 yields:

\[
AT^2 + BT + C = 0
\]  
(7)

in which A, B and C are:

\[
A = k_1 - k_2 + (1 + k_3) \cos(\delta_i - \alpha_i)
\]  
(8)

\[
B = -2 \sin(\delta_i - \alpha_i)
\]  
(9)

\[
C = k_1 + k_2 + (k_3 - 1) \cos(\delta_i - \alpha_i)
\]  
(10)

Thus, the value of \( \psi_i \) is given as:

\[
\psi_i = 2 \tan^{-1}\left( \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \right)
\]  
(11)

Finally, \( \theta_i \) can be derived from Eq. (1). In the above equations when \( r = 0 \) they turn into the ideal mechanism equations.

### 3.2 Direct Kinematics

The motion of the end-effector of the manipulator in Cartesian system is derived using direct kinematics, given the actuator variables. This problem is intractable in the presence of clearance because it is essentially a system of nonlinear equations for parallel manipulators. Neural Network has been utilized here for solving the direct kinematics problem. The flowchart of the proposed procedure has been depicted in Fig. 4. The majority of the analytical inverse kinematics solutions are used to train the Neural Network, and the remaining is utilized to test the trained network. We have 9 inputs, namely \((\theta_1, \gamma_1, \gamma_2, \gamma_3, \delta_1, \delta_2, \delta_3)\) in which \( \theta \) is a vector including \((\theta_1, \theta_2, \theta_3)\), and \((\gamma_1, \gamma_2, \gamma_3, \delta_1, \delta_2, \delta_3)\) are inferred from dynamic analysis.

![Fig. 4. The schematic of the Neural Network.](image)
3.3 Dynamic Analysis

As explained earlier, the clearance angles are known from the joint force directions. Dynamics of the manipulator is formulated to determine the joint forces. The joint rates, \( \dot{\mathbf{\theta}} = [\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3]^T \), are related to the end-effector Cartesian velocities, \( \mathbf{c} = [\dot{x}, \dot{y}, \dot{\phi}]^T \) by

\[
\dot{\theta} = J\mathbf{c}
\] (12)

where \( J \) is the Jacobian matrix and is defined as:

\[
J = \begin{bmatrix}
a_1/d_1 & b_1/d_1 & c_1/d_1 \\
a_2/d_2 & b_2/d_2 & c_2/d_2 \\
a_3/d_3 & b_3/d_3 & c_3/d_3
\end{bmatrix}
\] (13)

Furthermore we have:

\[
a_i = x - x_{oi} - l_i \cos \theta_i - l_i \cos \phi_i
\] (14)

\[
b_i = y - y_{oi} - l_i \sin \theta_i - l_i \sin \phi_i
\] (15)

\[
c_i = -l_i \left((y - y_{oi}) \cos \phi_i - (x - x_{oi}) \sin \phi_i \right) + l_{i3} \sin(\theta_i - \phi_i)
\] (16)

\[
d_i = l_i \left((y - y_{oi}) \cos \theta_i - (x - x_{oi}) \sin \theta_i \right) + l_{i3} \sin(\theta_i - \phi_i)
\] (17)

The time derivative of Eq. (12), leads to:

\[
\ddot{\theta} = J\ddot{\mathbf{c}} + \dot{J}\dot{\mathbf{c}}
\] (18)

It can be shown that the accelerations of the other links can be computed having those of the end-effector and the inputs. Following Newton-Euler equations we have 21 unknown variables appeared in 21 equations. They include 18 joint forces and 3 input torques. Fig. 5 demonstrates the force diagram (FBD) of the first leg. In this diagram \( T_1 \) indicates the input torque. Therefore, one can compute the clearance angles by [1,6,17]:

\[
\gamma_1 = \tan^{-1} \left( \frac{F_{y23}}{F_{x23}} \right), \quad \gamma_2 = \tan^{-1} \left( \frac{F'_{y23}}{F'_{x23}} \right), \quad \gamma_3 = \tan^{-1} \left( \frac{F''_{y23}}{F''_{x23}} \right)
\] (19)

\[
\delta_1 = \tan^{-1} \left( \frac{F_{y12}}{F_{x12}} \right), \quad \delta_2 = \tan^{-1} \left( \frac{F'_{y12}}{F'_{x12}} \right), \quad \delta_3 = \tan^{-1} \left( \frac{F''_{y12}}{F''_{x12}} \right)
\] (20)

in which \( F' \) and \( F'' \) are the joint forces of the legs 2 and 3, respectively.
CONTINUOUS PATH OPERATION

In continuous path operation, end-effector must exactly follow a specific path with predefined velocity and acceleration [26,27]. Given all points along the path, one can calculate the inputs from the inverse kinematics solutions in the absence of clearance \( r = 0 \), as explained in the inverse kinematics, in third section. Knowing \( \theta \), one can derive \( \dot{\theta} \) and \( \ddot{\theta} \) from Eqs. 12 and 18, and then calculate the acceleration of the other links. The angle of virtual links can be determined by Eqs. 19 and 20. In the absence of clearances, if we give the inputs \( \theta \) to the actuators, the end-effector follows the target trajectory precisely. In reality, however, the produced trajectory deviates from the desired one due to clearances. In order to find the inputs of the desired path, one should solve the inverse kinematics problem for real mechanism, i.e., with joint clearance. These inputs are named \( \theta_c \).

The real path of the end-effector is obtained from direct kinematics equations using \( \theta \) as input. Then, a Neural Network is utilized to efficiently solve the direct kinematics problem. In this case, 9 inputs \((\theta_1, \gamma_1, \gamma_2, \gamma_3, \delta_1, \delta_2, \delta_3)\) are given to the Neural Network and then the outputs are determined. Training the Neural Network is performed using samples taken from the solution of the inverse kinematics problem which is analytically tractable. In other words, we utilized \((\theta_c, \gamma_1, \gamma_2, \gamma_3, \delta_1, \delta_2, \delta_3)\) as the inputs to the networks and \((x, y, \phi)\) as the outputs to train the Neural Network. Fig. 6 shows the flowchart of the present method.

![Flowchart of the present method.](image)
5 RESULTS

The geometric and mass properties of the 3-RRR planar parallel manipulator are given in Table 1. Moreover, the end-effector of the manipulator is supposed to move with constant velocity of 0.1 m/s with constant orientation of $\phi = 0$ on a circular path with a radius of 0.18 m. We solved the problem for two instances; namely, vertical mechanism ($g=9.81$) and horizontal mechanism ($g=0$). It is noteworthy that the Neural Network is a feed forward network with three hidden layers. Table 2 summarizes the properties of the trained network.

Table 1. Geometric and mass properties of the manipulator

<table>
<thead>
<tr>
<th>$l_1$ (m)</th>
<th>$l_2$ (m)</th>
<th>$l_3$ (m)</th>
<th>$m_1$ (kg)</th>
<th>$m_2$ (kg)</th>
<th>$m_3$ (kg)</th>
<th>$r$ (clearance size)(mm)</th>
<th>Distance between motors (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4</td>
<td>0.35</td>
<td>0.2</td>
<td>1</td>
<td>0.75</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2. Neural Network parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Horizontal plane</th>
<th>Vertical plane</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum epochs</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Number of training data</td>
<td>119000</td>
<td>59500</td>
</tr>
<tr>
<td>Number of validation data</td>
<td>21000</td>
<td>10500</td>
</tr>
<tr>
<td>Stopping criterion</td>
<td>$10^{-7}$</td>
<td>$10^{-7}$</td>
</tr>
<tr>
<td>Number of hidden layer</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Number of neurons in each hidden layer</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>

The calculated virtual (clearance) angles from dynamic analysis of the horizontal and vertical planes are shown in Figs. 7 and 8, respectively. The clearance angle varies smoothly with time for the vertical plane. Nevertheless, few abrupt changes can be seen in the case of horizontal plane. This is because of the weight of the links. The clearance angles encounter extreme changes where the joint forces vanish. Therefore, it is more complicated to train the Neural Network in the case of the horizontal plane. Then we use more learning data to train the Neural Network.

![Fig. 7. Virtual link angles in the horizontal plane: (a) $\gamma_i$; (b) $\delta_i$.](image-url)
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Fig. 8. Virtual link angles in the vertical plane: (a) $\gamma_i$; (b) $\delta_i$.

The inputs for the desired path are calculated by inverse kinematics equations in the absence of clearance ($\theta$) and in the presence of clearance ($\theta_c$). Figs. 9 and 10 show the time variation of the actuator angles (inputs) for the mechanism in the horizontal and the vertical planes. Finding $\theta_c$ is of great importance, because this input is expected to exactly produce the desired path in the operation.

Fig. 9. Actuator angles for the mechanism in the horizontal plane, (a) $\theta_1$; (b) $\theta_2$; (c) $\theta_3$. 
Fig. 10. Actuator angles for the mechanism in the vertical plane, (a) $\theta_1$ ; (b) $\theta_2$ ; (c) $\theta_3$.

The desired and real paths of the end-effector are illustrated in Fig. 11(a). It is clear that the real path has been deviated from the desired path due to clearances. The other path namely “validating path” has been also superimposed into the graph. This trajectory is obtained using the already trained Neural Network but with a different population of $\theta_c$. In this way, we examine the accuracy of the algorithm. It can be seen in Fig. 11(a) that the validating path agrees considerably well with the desired path, providing some verification for the present method. The similar solution for vertical plane is depicted in Fig. 11(b), in which the real path slightly shifts downward with respect to the desired path. This is because of the weight of the links. In other words, the gravity forces in the vertical plane dominate the inertia forces.

In Figs. 12 to 14 the accuracy of the Neural Network with respect to three parameters, i.e., size of learning (training and validation) data, number of hidden layers and number of neurons in each layer are demonstrated. The accuracy is defined as the average of radial distances of the corresponding points taken from the desired and validating paths. As stated, the $\theta_c$ which is used to examine the accuracy of the Neural Network should be different from the $\theta_c$ which Neural Network used to train and validation. Moreover, each diagram shows the accuracy variation with respect to one parameter, while the other two are fixed. These values are given in Table 2. The accuracy diagrams show that the accuracy increases by increasing the learning data up to 70000 and 140000 data, respectively for the cases of the vertical and the horizontal planes. Therefore, one needs more data to train and examine the Neural Network for the case of horizontal plane. Moreover, the optimum values for the number of hidden layers and neurons are given in Figs. 13 and 14, respectively.
Fig. 11. Path of end-effector, (a) horizontal plane; (b) vertical plane.

Fig. 12. Accuracy of the Neural Network with respect to the number of hidden layers.

Fig. 13. Accuracy of the Neural Network with respect to the number of learning data.

Fig. 14. Accuracy of the Neural Network with respect to the number of neurons in each layer.
6 CONCLUSIONS

Continuous path operation of a 3-RRR planar parallel manipulator with clearance at the joints is investigated in this paper. Joint clearances have been treated as mass-less virtual links. The trajectories in the presence of clearances have been compared with those trajectories for the ideal case in the Cartesian and joint spaces. It has been studied for both vertical and horizontal planes. We have demonstrated that the variation of the clearances in the horizontal plane is more severe than in the vertical plane. This is because of the fact that the gravity can affect the dynamic behaviour of the manipulator in the vertical plane. In other words, the gravity forces dominate the inertia forces of the links. It is noteworthy that the clearance angles have been changed sharply whenever the joint forces vanish. Therefore, it has been more complicated to train the Neural Network in the case of the horizontal plane. So we have used more data to train and examine the Neural Network for this case. The accuracy diagrams of the Neural Network with respect to the three parameters have shown that by increasing the learning data the accuracy increases. Finally, our proposed method suggests that the errors caused by tolerances in the continuous path can be compensated via proper input transformation. This method provides a practical and cost effective approach to increase the positioning accuracy of this mechanism without changing the design. The developed procedure can also be applied for any manipulator with any running speed and payload.

REFERENCES


